

Piezoresistance and Piezo-Hall-Effect in *n*-Type Silicon

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Preliminary piezoresistance and piezo-Hall-effect measurements have been made on three relatively pure samples of *n*-type silicon over the temperature range 77–300°K to strains $\sim\frac{1}{2}\%$. The room-temperature value of the ratio of the saturation resistivity at large stress to the resistivity at zero stress confirms Long's estimate of the relative strength of the "across the zone face" umklapp scattering in intravalley acoustic phonon scattering, rather than a value an order of magnitude larger estimated by Dumke. Additionally, from the dependence of resistivity upon stress at 77°K, the value 8.3 ± 0.3 eV is found for the shear deformation potential tensor component Ξ_u . The dependence of both resistivity and Hall coefficient on stress at 77°K can be fitted quite well by theoretical expressions, deduced under certain reasonable simplifying assumptions, over the entire range of stress.

INTRODUCTION

PIEZORESISTANCE and piezo-Hall-effect measurements of a preliminary nature have been made on samples of relatively pure *n*-type silicon. The main object of the work was to resolve an uncertainty existing in the literature as to the relative importance of certain intervalley scattering mechanisms which play an important part in determining the conductivity relaxation time, τ , in this material. From Long's work on mobility as a function of temperature,¹ two important intervalley scattering mechanisms emerge (i) scattering from a given $\langle 100 \rangle$ valley to one of four equivalent valleys situated in approximately $\langle 110 \rangle$ directions from the first (*f* scattering), and (ii) scattering from the given valley along a $\langle 100 \rangle$ direction to the sixth valley of the set (*g* scattering). A phonon of characteristic temperature $T_{c1}\simeq 630^\circ\text{K}$ is believed to participate in *f* scattering, and a phonon with $T_{c2}\simeq 190^\circ\text{K}$ is thought to participate in *g* scattering. Both of these scatterings are "umklapp" transitions.

Long gives the following general expression² for the relaxation time for scattering of electrons by lattice vibrations only, as a function of electron energy ϵ ,

$$\tau_\alpha = \left\{ w_{A\alpha} (\epsilon/kT_0)^{1/2} (T/T_0) + \sum_i w_i (T_{ci}/T_0)^{3/2} \right. \\ \left. \times \left[\frac{(\epsilon/kT_{ci}+1)^{1/2}}{\exp(T_{ci}/T)-1} + \frac{(\epsilon/kT_{ci}-1)^{1/2} \text{ or } 0}{1-\exp(-T_{ci}/T)} \right] \right\}^{-1}. \quad (1)$$

The subscript α stands for either \parallel or \perp , giving the two principal components of the relaxation time tensor. The w 's are coupling constants, $w_{A\alpha}$ being for acoustic (or intervalley) scattering and the w_i for intervalley processes. The constant T_0 is so chosen that $w_{A\alpha}$ has no

temperature dependence. For the case of two intervalley processes only, (1) abbreviates to the form,

$$\tau_\alpha^{-1} = w_{A\alpha} C_0 + w_1 C_1 + w_2 C_2, \quad (2)$$

where the second and third terms represent *f* and *g* scattering, respectively. Long finds the following relative values of the coupling constants:

$$w_{A11}/w_{A1} = 1.5; \quad w_1/w_{A1} = 2.0; \quad w_2/w_{A1} = 0.15, \quad (3)$$

by a detailed fit to the temperature variation of conductivity and Hall constant data. With these values of the w 's, it can be readily shown from (1) that at room temperature, acoustic, *f*, and *g* scattering processes contribute to $1/\tau$ in the approximate proportions, 38%, 53%, and 9% (ignoring for the present the anisotropy of acoustic scattering). Dumke's calculations,³ based on data on the low-temperature recombination radiation of Si, lead to the relative proportions 22, 22, and 56%. It is this rather violent disagreement (in particular, the order-of-magnitude difference in the estimates of the importance of *g* scattering in relation to acoustic scattering) which largely motivated the present study.

By applying an axial compressive stress along a $\langle 100 \rangle$ direction, the band edge for the two "parallel" spheroids (those with rotation axis along the stress direction) is lowered in relation to the band edge for the four "perpendicular" spheroids, and electrons from the latter consequently empty into the former. When the relative levels of the band edges have been separated by an amount $\gg kT$, the material becomes essentially a new semiconductor with electrons located in the two parallel spheroids only. Since *f* scattering from the two occupied spheroids is then completely eliminated, the second term on the right-hand side of (2) drops out; hence, an assessment can more readily be made of the importance of the *g* scattering term.

In the preliminary work reported here, three samples, referred to as Si 1, 2, and 3 were studied. Their characteristic parameters are listed in Table I. Each sample was cut in the form of a rectangular parallelepiped of approximate dimensions 1.2 mm \times 1.2 mm \times 10 mm,

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¹ D. Long, Phys. Rev. **120**, 2024 (1960).

² See also, C. Herring, Bell System Tech. J. **34**, 237 (1955).

³ W. P. Dumke, Phys. Rev. **118**, 938 (1960).

TABLE I. Pertinent parameters of samples.

Sample	Zero-stress resistivity, Ω cm			Zero-stress Hall data; $(R_H e)^{-1} \text{ cm}^{-3} \times 10^{-18}$		
	Room temp.	190°K	77°K	Room temp.	190°K	77°K
Si 1	173	...	18.2			
Si 2	151	61	17.4			
Si 3	182	75	26.5	2.70	2.84	2.77

with $\langle 100 \rangle$ directions accurately aligned along the edges. The samples were lapped and ground, taking care that the ends were accurately parallel in order to reduce the possibility of buckling when the stress was applied. (The maximum stress amounted to about 8 tons per cm^2 , corresponding to a strain of 0.5%.) Electrical contacts were made by first depositing nickel on the sample surfaces by the electroless technique described by Sullivan and Eigler,⁴ then alloying small grains of tin with the nickel, and finally attaching platinum leads to the tinned areas. Excess nickel was subsequently removed by etching. Measurements of resistivity versus stress were made on all three samples in the neighborhood of three fixed temperatures, namely, room temperature, solid CO_2 -alcohol temperature ($\sim 190^\circ\text{K}$), and liquid nitrogen temperature ($\sim 77^\circ\text{K}$); in addition, the dependence of the Hall coefficient upon stress was studied for Si 3 at the three temperatures.

RESISTIVITY AS A FUNCTION OF STRESS

The conductivity of a sample in a $\langle 100 \rangle$ direction is given by

$$\sigma = n_{11} e \mu_{11} + 2n_{\perp} e \mu_{\perp}, \quad (4)$$

where n_{11} is the number of electrons in the pair of parallel spheroids and n_{\perp} is the number of electrons in a pair of perpendicular spheroids; μ_{11} and μ_{\perp} are the respective mobilities in the two types of spheroids for conduction in this direction. The application of an axial

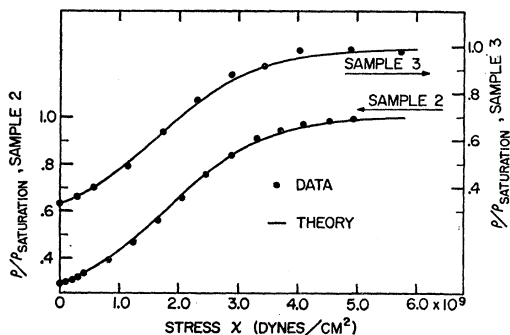


FIG. 1. Experimental variation of the resistivity of samples Si 2 and Si 3 at 77°K with stress χ , in units of their respective saturation stress. The solid curve is a plot of Eq. (9) with $\beta = 1.25 \times 10^{-9} \text{ cm}^2/\text{dyn}$ and K determined using Eq. (8).

⁴ M. V. Sullivan and J. H. Eigler, J. Electrochem. Soc. **104**, 226 (1957).

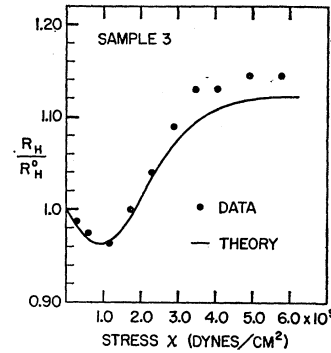


FIG. 2. Experimental variation of the Hall constant R_H of sample Si 3 at 77°K with stress χ , in units of the zero stress Hall constant R_H^0 . The solid curve is a plot of Eq. (11) with $\beta = 1.25 \times 10^{-9} \text{ cm}^2/\text{dyn}$ and $K = 3.96$ as determined using Eq. (8).

compressive stress χ shifts the band edges for the parallel and perpendicular valleys by a relative amount $\Delta\epsilon$; for the case of Boltzmann statistics

$$n_{\perp}/n_{11} = \exp(-\Delta\epsilon/kT) \equiv \exp(-\beta\chi); \quad (5)$$

$$\beta = \Xi_u(S_{11} - S_{12})/kT,$$

where Ξ_u is a component of the deformation potential tensor^{2,5,6} and S_{11} , S_{12} are elastic compliance constants. We shall first be concerned with the information obtainable from the ratios of sample resistivity at zero stress, ρ_0 , to the value in the limit $\chi \rightarrow \infty$, ρ_s (the saturation resistivity). Inserting first $n_{11} = n_{\perp} = N/3$ and secondly $n_{11} = N$, $n_{\perp} = 0$ into (4), one obtains

$$(\rho_s/\rho_0) = (\sigma_0/\sigma_s) = (\mu_{11}^0 + 2\mu_{\perp}^0)/3\mu_{11}^s$$

$$= \frac{1}{3}(\tau_{11}^0/\tau_{11}^s + 2\tau_{\perp}^0 m_{11}/\tau_{11}^s m_{\perp}). \quad (6)$$

The effective mass components do not change⁷ with χ , though, of course, τ may. Using the cyclotron resonance mass values⁸ $m_{11} = 0.90m_0$, $m_{\perp} = 0.192m_0$, (6) takes the form,

$$(\rho_s/\rho_0) = \frac{1}{3}(\tau_{11}^0/\tau_{11}^s + 9.4\tau_{\perp}^0/\tau_{11}^s). \quad (7)$$

At 77°K , the f intervalley mechanism, according to both Dumke³ and Long,¹ may be neglected; if the remaining scattering mechanisms are taken to be independent of stress, then $\tau_{11}^0 = \tau_{11}^s$ and (7) further simplifies to

$$(\rho_s/\rho_0) = (1 + 9.4t)/3; \quad t = (\tau_{\perp}/\tau_{11}). \quad (8)$$

TABLE II. Ratio of resistivity at saturation stress to the zero-stress resistivity.

	Si 1	Si 2	Si 3
ρ_s/ρ_0	3.7	3.4	3.0
$t = \tau_{\perp}/\tau_{11}$	1.08	0.98	0.85

⁵ E. N. Adams, Chicago Midway Laboratory Technical Report CML-TN-P8 (unpublished).

⁶ The sign of Ξ_u , of course, determines whether the parallel valleys move up or down in energy with respect to the others for a given sign of stress. The sign of Ξ_u , inadvertently stated as negative by Herring and Vogt (reference 12, below), has been shown to be positive by D. K. Wilson and G. Feher, Phys. Rev. **124**, 1028 (1961).

⁷ J. C. Hensel and G. Feher, Phys. Rev. Letters **5**, 307 (1960).

⁸ C. J. Rauch, J. J. Stickler, H. J. Zeiger, and G. S. Heller Phys. Rev. Letters **4**, 64 (1960).

The anisotropy of the relaxation time as given by t can, therefore, be found from the observed value of (ρ_s/ρ_0) for the three samples at 77°K; the results are given in Table II. It is evident that impurity scattering is significant at this temperature in these samples; by comparing the data of Table II with the recent results of Neuringer and Little⁹ it is clear that the samples must be highly compensated.

We next consider the situation at room temperature. Here the results indicate that impurity scattering can be ignored so that the principal components of the relaxation time tensor are given by (2). The τ 's which occur in Eq. (7) can be calculated using either Long's¹ or Dumke's³ results and the calculated values of (ρ_s/ρ_0) can be compared with experiment. The results¹⁰ are given in Table III. It is evident that the experimental value accords much better with Long's results than with Dumke's, and we may conclude that g scattering is relatively weak at room temperature. The results found at 190°K confirm qualitatively a low value of the ratio w_2/w_{A1} but at this temperature impurity scattering is significant and Eq. (2) can no longer be used to calculate τ 's and thereby permit a quantitative comparison to be made.

For the assumptions under which (8) is valid, the dependence of ρ on χ can easily be shown to have the form

$$(\rho/\rho_0) = \frac{(1+2K)[1+2\exp(-\beta\chi)]}{3[1+2K\exp(-\beta\chi)]}, \quad K = \mu_l/\mu_{11}. \quad (9)$$

$$R_H = \frac{\langle\tau\rangle^2 K [1+2\exp(-\beta\chi)] [1+(K+1)\exp(-\beta\chi)]}{Ne\langle\tau^2\rangle [1+2K\exp(-\beta\chi)] [K+(K+1)\exp(-\beta\chi)]}. \quad (10)$$

In the limits $\chi=0$ and $\chi \rightarrow \infty$, (10) reduces to the familiar forms²

$$R_H^0 = \frac{\langle\tau\rangle^2}{Ne\langle\tau^2\rangle} \frac{3K(K+2)}{(1+2K)^2}, \quad R_H^\infty = \frac{\langle\tau\rangle^2}{Ne\langle\tau^2\rangle}.$$

If τ is assumed to be independent of χ (an assumption discussed above), then

$$\frac{R_H}{R_H^0} = \frac{(1+2K)^2 [1+2\exp(-\beta\chi)] [1+(K+1)\exp(-\beta\chi)]}{3(2+K) [1+2K\exp(-\beta\chi)] [K+(K+1)\exp(-\beta\chi)]}. \quad (11)$$

R_H/R_H^0 has been computed from (11), using the values $K=3.96$ and $\beta=1.25 \times 10^{-9}$ found before, and is compared with experiment (Si 3 at 77°K) in Fig. 2. The fit is clearly very satisfactory for $\chi \leq 2 \times 10^9$, but at higher χ , the experimental points diverge from the theoretical

⁹ L. J. Neuringer and W. J. Little, in *Proceedings of the 1962 International Conference on the Physics of Semiconductors, Exeter*, edited by A. C. Stickland (The Institute of Physics and The Physical Society, London, 1962); Raytheon Tech. Memorandum T-381, 1962 (unpublished).

¹⁰ These results were obtained with the aid of a photographic enlargement of Herring's (reference 2) Fig. 4 which gives the computed integrals necessary for our Eq. (1).

TABLE III. Comparison of the experimental saturation to zero-stress resistivity ratio at 290°K with the values implied in the works of Long and Dumke.

ρ_s/ρ_0	Calculated value		Experimental value ^c
	Long ^a	Dumke ^b	
	2.3	3.0	2.36±0.3

^a Reference 1.

^b Reference 3.

^c This is the mean of the results for Si 1 and Si 3; Si 2 fractured prematurely.

This expression has been fitted to the experimental curve of (ρ/ρ_0) versus χ for Si 2 and 3 at 77°K. The result is shown in Fig. 1. K was first found from (ρ_s/ρ_0) ; from the fitting procedure, β is 1.25×10^{-9} cm²/dyn. From this and the tabulated values¹¹ of S_{11} and S_{12} , one obtains $\Xi_u = 8.5$ eV. The data for sample 1 give a poorer fit to (9) and yields a value $\Xi_u = 8.0 \pm 0.4$. A reasonably weighted average of the present results gives $\Xi_u = 8.3 \pm 0.3$ eV. This falls well within the (corrected) range 8.1–9.2 eV given by Herring and Vogt¹² and agrees with the value 8.1 eV which may be extracted from Long's Eq. (7) and his subsequent discussion. Less accurate values for Ξ_u have also been given by Wilson and Feher⁶ (11 eV) and Castner¹³ (8.0–10.0 eV).

HALL COEFFICIENT AS A FUNCTION OF STRESS

The dependence of the Hall Coefficient, R_H , upon stress has the form¹⁴

curve by $\sim 3\%$. The fit at higher χ can be improved by adopting a value of K of the order of 4.5, but then the close agreement between experiment and theory at low χ is lost, the departure again amounting to $\sim 3\%$ (at $\chi \approx 10^9$). Since Si 3 has a significant amount of impurity scattering at 77°K, this small but real discrepancy may be due to impurity induced intervalley

¹¹ H. J. McSkimin, *J. Appl. Phys.* **24**, 988 (1953).

¹² C. Herring and E. Vogt, *Phys. Rev.* **101**, 944 (1956). In particular, cf., *ibid.* **105**, 1933 (1957).

¹³ T. G. Castner, Jr., *Phys. Rev. Letters* **8**, 13 (1962).

¹⁴ Cf., R. A. Smith, *Semiconductors* (Cambridge University Press, Cambridge, 1959), p. 121.

scattering¹⁵ at the lower strains; such scattering has been ignored in deriving (11).

CONCLUSIONS

The preliminary results presented here show quite clearly that the matrix element for "across the zone face" scattering in silicon is quite small and is consistent with the results of Long.¹ Both Long's results and the

¹⁵ G. Weinreich, T. M. Sanders, Jr., and H. G. White, *Phys. Rev.* **114**, 33 (1959).

present ones assume a temperature-independent effective mass; if, as suggested,¹⁶ the effective mass increases with temperature, the effect would be to make this scattering rate even less.

As an ancillary result, a value of 8.3 ± 0.3 eV has been obtained for Ξ_{uc} , the shear deformation potential of the conduction band of silicon.

¹⁶ M. Cardona, W. Paul, and H. Brooks, *Helv. Phys. Acta* **33**, 329 (1960).

Polarization Effects in the Magnetic Elastic Scattering of Slow Neutrons*

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The theory of the elastic scattering of polarized neutrons by magnetic crystals with ordered spins is developed. Several terms which were omitted by Halpern and Johnson in their original treatment of the subject are discussed. These terms vanish in the case of scattering from simple ferromagnetic or antiferromagnetic structures, but they give rise to some interesting effects in more complex structures. Among these is a polarization effect which occurs in antiferromagnetic spirals, proposed recently by Overhauser, Nagamiya, and Izyumov, and an effect which allows the determination of the imaginary part of the form factor in noncentrosymmetric systems. General formulas for the cross section and for the polarization of the scattered beam are given for arbitrary spin orderings.

INTRODUCTION

POLARIZED neutron scattering has, in the past few years, proved a most useful technique for the study of the magnetic properties of solids. The theory of the scattering of polarized beams was developed by Halpern and Johnson¹ in their now classic paper on the magnetic scattering of slow neutrons, and the expressions which they derived have been verified experimentally.^{2,3} In their derivations they restricted their attention to the cases of ferromagnets and simple antiferromagnets. As a result of this restriction they omitted several terms which should appear in the cross section for scattering of a polarized beam and in the expression for the polarization of the scattered beam. These terms are of interest in view of the complicated and unusual spin arrangements found in the last few years. Two of these terms give rise to an interesting polarization effect in the case of scattering by spiral spin structures, as proposed recently by Overhauser,⁴

Nagamiya, and Izyumov⁵ while another is of interest in connection with the imaginary part of the magnetic form factor.

In this paper complete expressions for the elastic scattering cross section of a system of ordered spins are derived, along with relations for the polarization of the scattered beam. Several examples are presented to illustrate the occurrence of the terms omitted by Halpern and Johnson. In order to derive the formulas we make use of the density matrix description of the polarized beam. This was first used by Tolhoek and de Groot⁶ and Wolfenstein⁷ in the case of nuclear scattering, and was applied to the diffraction problem by Marshall.⁸ Particularly clear descriptions of this useful concept are given by Fano and by ter Haar.⁹

The principal expressions derived are Eq. (15) for the cross section for elastic scattering of a polarized beam by ordered spins and Eq. (19) for the final polarization of a beam elastically scattered by ordered spins. Equations (8) and (17) are more general and may be

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¹ O. Halpern and M. H. Johnson, *Phys. Rev.* **55**, 898 (1939).

² C. G. Shull, E. O. Wollan, and W. C. Koehler, *Phys. Rev.* **84**, 912 (1951).

³ R. Nathans, C. G. Shull, G. Shirane, and A. Andresen, *J. Phys. Chem. Solids* **10**, 138 (1959); R. Nathans, T. Riste, G. Shirane, and C. G. Shull (unpublished).

⁴ A. W. Overhauser, *Bull. Am. Phys. Soc.* **7**, 241 (1962).

⁵ T. Nagamiya (private communication to R. Nathans); Yu. A. Izyumov, *Zh. Eksperim. i Teor. Fiz.* **42**, 1673 (1962) [translation: *Soviet Phys.—JETP* **15**, 1162 (1962)].

⁶ H. A. Tolhoek and S. R. de Groot, *Physica* **15**, 833 (1949).

⁷ L. Wolfenstein, *Phys. Rev.* **75**, 1644 (1949).

⁸ W. Marshall, *Lectures on Neutron Diffraction*, Harvard, 1959 (unpublished).

⁹ U. Fano, *Rev. Mod. Phys.* **29**, 74 (1957); D. ter Haar, *Rept. Progr. Phys.* **24**, 304 (1961).